# The modeling and determination of dynamic elastic modulus of magnesium based metal matrix composites

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The aim of the present study was to determine elastic modulus of the magnesium-based composites containing different volume fraction of SiC particulates using an innovative free-free beam type impact based technique. This technique is based on classical vibration theory, by which the geometry and material properties of the metal matrix composites are related to resonant frequency of the test specimen. With the fundamental resonant frequency obtained from the experiment and density determined by the Archimedes' principle, the elastic modulus values were determined. In addition, a finite element model is proposed for different SiC weight percentage samples for the determination of dynamic elastic modulus using the first natural frequency corresponding to the flexural mode. The elastic modulus values obtained using finite element method were found to be in close agreement with the values obtained from the impact based experiments and in better agreement when compared to theoretical methods such as Halpin-Tsai method. Both the theoretical approaches, in common, exhibit the increasing trend of elastic modulus value with an increase in weight percentage of SiC particulates. 

## 1. Introduction

Metal Matrix Composites (MMCs) represent the unified combination of stronger and stiffer ceramic reinforcement with ductile and tougher metallic materials [1]. The properties of these composites can be tailored to specific design requirements. The type of properties which can be controlled in the final composites are for example, stiffness, density, thermal expansion coefficient, wear resistance, hardness, and strength [2–6]. As such MMCs have the potential to serve to a spectrum of applications provided a judicious selection of metallic matrix, ceramic reinforcement, processing technique and heat treatment procedure is made. The result of existing literature search shows that the MMCs are actively being pursued as potential candidate materials in aerospace, automotive, electronics and sports sector [1].

Amongst the various mechanical properties, stiffness or elastic modulus of the metallic matrix can be considerably enhanced as a result of addition of ceramic reinforcement. The realization of enhanced stiffness is particularly desirable for the materials that have to be used for the fabrication of components designed primarily based on stiffness. In stiffness based design the constraint is to minimize the deflection rather than stress, hence the young's modulus has to be controlled. In addition, if the design requires minimizing the mass and maximizing the stiffness of the system, then the designer aims to select a material with higher specific stiffness ($E/\rho$), where $E$ and $\rho$ are the elastic modulus and density, respectively. The quantity $(E/\rho)^{1/2}$ also represents the velocity of the elastic waves. Thus, aiming for higher specific stiffness results for higher natural frequency for the component. New materials such as magnesium-silicon carbide (Mg/SiC) based composites have been developed with higher stiffness and lower density to suit to the needs in designing dynamic systems. This has been possible by controlling the reinforcement percentage in the metal matrix. Reinforcement of the magnesium matrix with SiC has distinct advantage due to [5]:

- Relatively low cost and ready availability of SiC.
- High modulus and strength of SiC.
- Resultant MMC density only slightly higher than magnesium.
- SiC is thermodynamically stable in molten magnesium.

Elastic modulus can be determined by both static and dynamic methods. The terms static and dynamic are interpreted in terms of strain rate and strain amplitude. Static testing such as tensile test involves relatively large elastic strains and slow strain rates, while dynamic testing methods such as continuous excitation and impulse excitation involves small strains (on the order of $10^{-6}$) and high strain rates [7]. Generally, dynamic modulus of materials is a function of frequency and at lower frequency, flexural beam methods are used to determine the flexural natural frequencies [8], while at high frequencies the speed of ultrasonic waves through the medium is used to compute the dynamic modulus [9]. Continuous excitation and impulse excitation can measure the dynamic elastic modulus but differ in the manner in which the specimen are vibrated or excited. Under continuous excitation the specimen is continuously vibrated and the operating frequency is varied to check for resonance. The impulse excitation technique [10, 11], is accomplished by exciting all the resonance frequencies of the specimen corresponding to a direction using a force sensor attached hammer and the vibration is measured using a small mass type accelerometer or a laser vibrometer. The impulse excitation method combines rapid testing, high accuracy and simplicity when using appropriate test specimen geometry such as a cylindrical rod.

The dynamic modulus studies of several composite materials including MMCs have been studied by various researchers using methods similar to piezoelectric ultrasonic composite oscillator technique (PUCOT) and compared with modulus values obtained from static techniques [9]. It was observed that the modulus values measured were more accurate by dynamic means than by static means and that the static moduli scattered about the dynamic values. The literature search, however, did not reveal any systematic studies on the determination of dynamic modulus of MMCs using impact based methods.

Accordingly, in the present study, an impact method (free-free beam method) was adopted to measure dynamic elastic modulus of SiC reinforced extruded magnesium specimens and to study the variation of elastic modulus with different weight percentage of reinforcement. The dynamic modulus was calculated using the lowest natural frequency corresponding to the bending type deformation mode, acquired from the vibration response of the composite rod to a known impulse load (also called modal analysis). Further, finite element method (FEM) was used to theoretically predict the dynamic modulus of the material by modeling long slender round bars with different volume percentages of SiC particulates in the magnesium matrix.

## 2. Theoretical considerations

The flexural vibration of a long slender specimen in the transverse direction can be mathematically described by the Bernoulli-Euler equation [12].

$$EI.\frac{d^4y}{dx^4} - \rho A\omega^2 y = 0 \tag{1}$$

where, $y$ is the transverse displacement, $\omega$ is the frequency, $x$ is the position along the beam, $\rho$ is the density, $A$ is the cross-sectional area of the beam, $E$ is the Young's modulus and $I$ is the second moment of the cross-sectional area.

Equation 1 is a fourth order differential equation, which can be solved to arrive at the natural frequency of vibration of the beam for various end support conditions [8, 12]. For example, under the free-free support condition, the beam is suspended at the vibration nodal points by nylon strings and the corresponding natural frequency of the beam is given by Equation 2.

$$\omega_n = 22.4\sqrt{\frac{EI}{A\rho l^4}} \tag{2}$$

Using Equation 2 the dynamic elastic modulus can be expressed in terms of the natural frequency, material density and the beam dimensions, viz., diameter ($d$) and length ($l$) as follows:

$$E = \frac{\omega_n^2 \rho l^4}{31.36d^2} \tag{3}$$

## 3. Finite element model

Finite Element Method (FEM) is a viable method to analyze particulate dispersed metal matrix composites due to its modeling capability of the particulate. FEM is one of the tools, which has the distinct advantage to model more closely to the exact geometry. Secondly, it has the advantage to model the various material properties in the geometry as well as the anisotropy of the individual phases. Thirdly, it can model the exact fixity and the load conditions on the specimen [13].

Mostly FEM has been successfully used in analyzing laminates or fiber reinforced composites. In the present work, the particulates have been assumed to have cubic morphology with unit aspect ratio. The particulates were positioned in the matrix with uniform spacing in radial, angular and axial directions so as to achieve global isotropic conditions (see Fig. 1). The size of the particulate was calculated based on the volume fraction of the reinforcement.

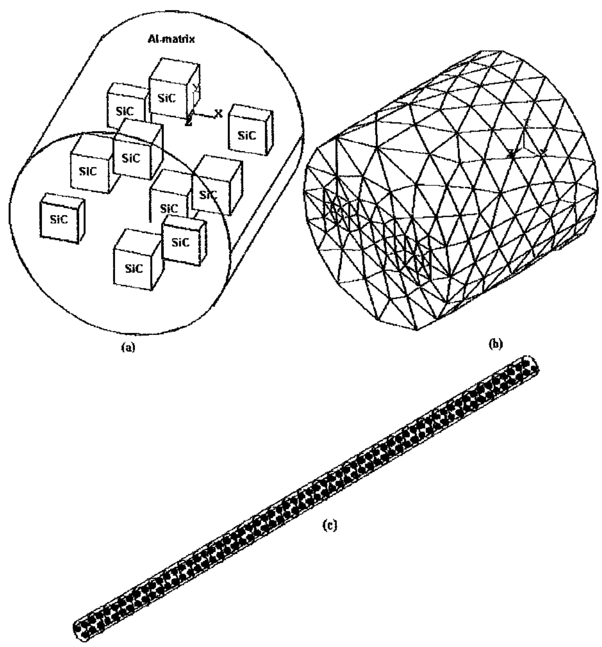


*Figure 1* Placement of the SiC particulates in the FEM model of the beam. (a) Solid model showing the SiC particulates in a unit cell ($l = 10$ mm, $d = 10$ mm). (b) Corresponding FEM model of the unit cell. (c) Solid model of a long slender beam ($l = 300$ mm, $d = 10$ mm) by repetitive placement of the unit cells.

A three dimensional model of a circular MMC rod was built using the ANSYS5.5 FEM software [14], with a dimension of 10 mm diameter and 300 mm length, similar to the dimension of the actual sample under experimental study (see Fig. 1). Eight cube shaped SiC particles with unit aspect ratio were placed at equal distance in the radial, axial and circumferential directions in 10 mm length of the rod model. The particle shape was assumed to be square since it closely matches with the sharp faceted nature of the SiC particles used in the experimental sample. From the analysis, it was found that the final bending stiffness of the FEM based MMC beam model highly depends on the dispersion characteristics of the reinforcement viz., position of the SiC square particle from the neutral axis of bending. In the present model, the particle was placed at the mid-position along the radial direction and was maintained for all the different volume fraction of reinforcement. The size of the cubic particles was varied to suit the various volume fraction of the reinforcement in the unit cell. This method was contrary to the actual sample where the reinforcement percentage is increased by adding more number of same-sized particles. This was mainly because in the present model we have fixed the overall dimensions of the beam model. Hence, placement of increased number of smaller particles inside the beam was difficult to achieve for higher volume fraction because it leads to physical contact of the particles and ends up in forming a continuous fiber reinforced type FEM model. It may be noted here that during modeling of a smooth faceted particle shape such as a sphere or ellipsoid, as well as, in the case of increased number of small particle dispersion, the FEM model ends up with high mesh count, which exceeds a typical Pentium™ computer's performance limits. The FEM model assumes the interface between the reinforcement particle and the matrix to have perfect adherence. The elastic modulus of magnesium [15] and SiC [16] was taken to be 45 GPa and 450 GPa, respectively, and the density was taken to be 1740 $Kg/m^3$ and 3200 $Kg/m^3$, respectively.

The linear dynamic equation of the MMC rod can be given based on force balance principle.

$$[M] \cdot \{y\}^{\bullet\bullet} + [B] \cdot \{y\}^{\bullet} + [K] \cdot \{y\} = \{f\} \qquad (4)$$

where, $[M]$ = the mass matrix, $[B]$ = the damping matrix, $[K]$ = the stiffness matrix, $\{y\}$ = the displacement vector, $\{f\}$ = the applied force vector, $t$ = time, $\{y\}^{\bullet\bullet} = \mathrm{d}^2y/\mathrm{d}t^2$, $\{y\}^{\bullet} = \mathrm{d}y/\mathrm{d}t$.

The mass, stiffness and damping matrices are calculated in the computer program based on the input elastic modulus, Poisson's ratio, density, and damping coefficient of the reinforcement phase and the metallic matrix. The overall dynamic behavior of the MMC material depends on the properties of the individual phases and the dispersion characteristics of the reinforcement phase. Hence the FEM model should be in close accordance with the actual material in a global perspective.

Under no load conditions, the natural frequencies and mode shapes, also called eigen values and eigen vector, under zero damping can be computed based on typical eigen value solver such as Subspace iteration method, Simultaneous iteration method or Block Lanczos method. In this analysis Block Lanczos method [14] was used to extract the natural frequencies and mode shapes for the MMC beam model. Based on the mass of the beam the density was estimated. Using Equation 3 the equivalent elastic modulus of the MMC was evaluated. Further an empirical relation was fit, see Equation 5, between the FEM predicted modulus, $E_{FEM}$, and the reinforcement weight percentage, $W$, of the MMC, using MATLAB software. Using Equation 5 the dynamic elastic modulus corresponding to the experimental reinforcement's weight percentage was calculated and is listed in Table I.

$$E_{FEM} = 45 + 0.1955\,W^2 - 2.1222e\text{-}2\,W^3 + 8.8489e\text{-}4\,W^3 - 1.233e\text{-}5\,W^5 \qquad (5)$$

TABLE I Results of the theoretical and experimental determination of dynamic elastic modulus values for Mg-SiC composite samples

| Weight % | Density ($g/cm^3$) | Porosity (%) | Free-Free method, $E_{free}$ (GPa) | Halpin-Tsai method, $E_{H\text{-}T}$ (GPa) | FEM method, $E_{FEM}$ (GPa) |
|---|---|---|---|---|---|
| 0.0* | 1.74 | 0.00 | 45.00 | 45.00 | 45.00 |
| 7.6 | 1.78 | 0.84 | 45.04 | 51.37 | 49.62 |
| 14.9 | 1.85 | 1.14 | 49.38 | 58.35 | 52.78 |
| 26.0 | 1.96 | 0.91 | 54.09 | 70.85 | 62.04 |

*The properties corresponding to the monolithic magnesium sample are taken from reference [15].

## 4. Experimental procedures

### 4.1. Materials

In this study, magnesium based composites containing 7.6, 14.9 and 26.0 weight percentages of SiC particulates processed using molten metal based technique and hot extruded at 350°C were analysed. Fig. 2 shows a typical SiC particulate distribution pattern in the Mg-SiC MMC rod with 7.6 weight percentage of SiC.

### 4.2. Density and porosity measurement

The densities of the extruded composite samples were measured by Archimedes' principle [17]. The specimens were weighed in air and in distilled water using an A & D ER-182A electronic balance to an accuracy of

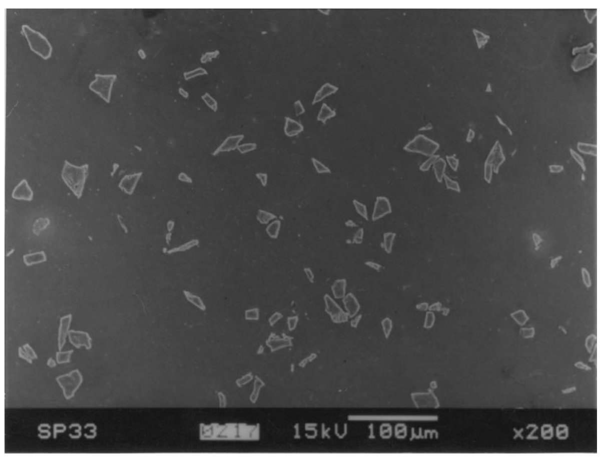


*Figure 2* Representative SEM micrograph showing uniform distribution of SiC particulates observed in Mg-SiC MMC rod with 7.6 weight percentage of SiC.

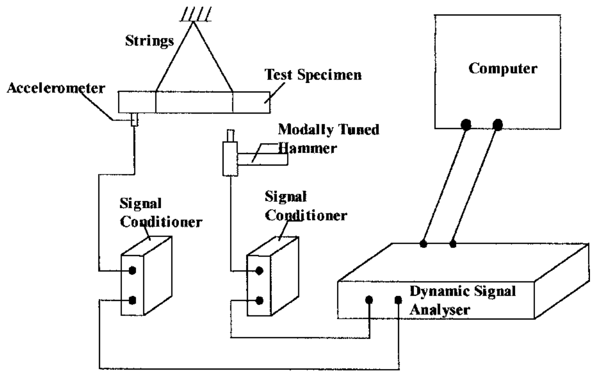


*Figure 3* Schematic diagram showing experimental setup of Free-Free beam technique.

±0.0001 g. The porosity was then calculated from the measured composite density, theoretically computed rule of mixture density and the reinforcement weight fractions [17].

### 4.3. Free-free method

The most popular flexural mounting method is Free-Free beam method (see Fig. 3). Based on the ASTM C1259-96 standard [10], the beam is suspended at its nodes by two nylon strings whose inertia is very low. Since it is virtually impossible to find the exact location of the nodes, some energy is inevitably lost in the vibration of the strings. Under a resonance condition, nodes and anti-nodes are the locations in the beam that would undergo minimum and maximum vibration amplitude, respectively, and can be detected from the mode shape.

The experiment was performed with the accelerometer placed at the anti-nodal position [12], viz., at the end or at the center of the beam length so as to capture the first mode of flexural vibration, which would be similar to a half sine wave. Initially a preliminary analysis was performed to find the nodal points of the bending mode. The strings were placed at this position to minimize the energy loss passed into the strings. The nodal points for beams of uniform section in a free-free suspension are at distances from the free ends of approximately $0.22L$ and $0.78L$, where $L$ is the length of the beam [18]. Very little exciting energy is required, even for very large specimens, because the measurement is performed at very low strain amplitude. Hence only a very light tap is sufficient to initiate the measurement.

The excitations were provided using a modally tuned hammer, which has a load sensor at the tip. The tip material can be changed from nylon to steel so as to change the impulse duration when the excitation is applied [12]. Following excitation, first mode of flexural vibrations was captured by accelerometer and the frequency response function (FRF) thus obtained was used to determine natural frequency. Typical frequency plots for the three composite samples are shown in the Fig. 4a to c. The natural frequency ($\omega_n$) thus obtained

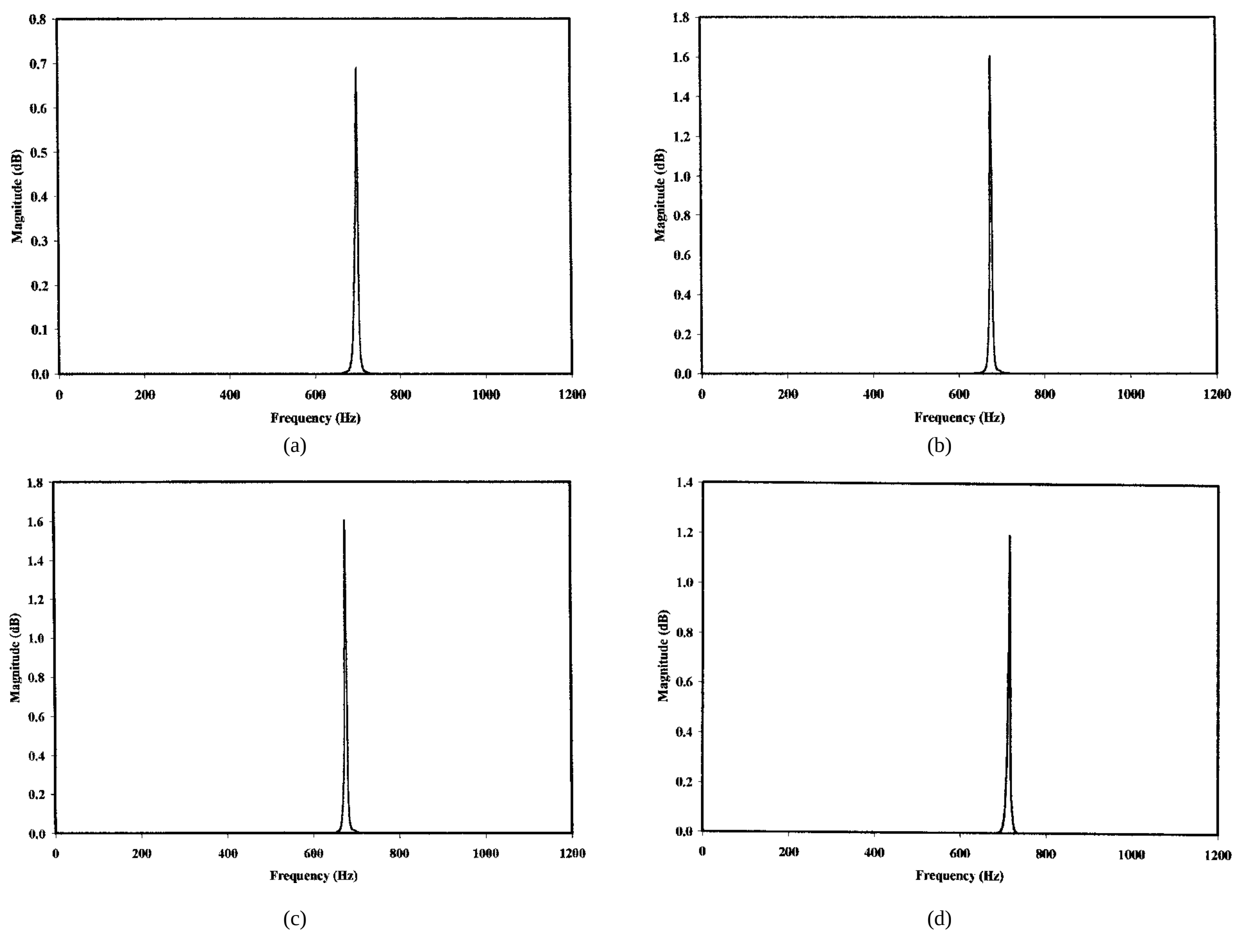


*Figure 4* (a) Typical FRF plot for the pure magnesium rod. (beam length: 249 mm and beam diameter: 9.83 mm (b) Typical FRF plot for the Mg-SiC MMC rod with 7.6 weight % of SiC. (beam length: 254 mm and beam diameter: 9.82 mm) (c) Typical FRF plot for the Mg-SiC MMC rod with 14.9 weight % of SiC. (beam length: 255 mm and beam diameter: 9.54 mm) (d) Typical FRF plot for the Mg-SiC MMC rod with 26.0 weight % of SiC. (beam length: 254 mm and beam diameter: 9.85 mm).

was used to determine the dynamic elastic modulus using Equation 3.

## 5. Results

### 5.1. FEM predictions vs. experimental measurements

The influence of the SiC reinforcement on the dynamic elastic modulus of the metallic matrix was investigated by modeling the uniformly distributed cubic reinforcement particulates in a circular beam of 300 mm length and 10 mm diameter with various volume percentage of SiC particulates, as shown in Fig. 1. The dynamic elastic modulus $E_{FEM}$ listed in Table I for each of the composite samples was computed using Equation 3 assuming perfect isotropic conditions.

The elastic modulus measurement of composite samples containing three different weight percentage of reinforcement viz., 7.6%, 14.9% and 26.0%, was performed separately and benchmarked against the pure magnesium sample. The dynamic elastic modulus, $E_{free}$, computed using the extracted first modal frequency using Equation 3 for the composite samples are listed in Table I. Fig. 5 shows the variation of the elastic modulus with the weight percentage, from both experimental and FEM predictions.

Among the various theoretical methods [19] to predict the elastic modulus of the particulate dispersed MMCs, Halpin-Tsai equation is found to be closer to the experimental results. Table I lists the elastic modulus predicted using Halpin-Tsai method (Equation 6) for a typical unit aspect ratio reinforcement particulate.

$$E_{H\text{-}T} = \frac{E_m(1 + 2sqV_p)}{1 - qV_p} \quad (6)$$

where, $E_p$ and $E_m$ are the elastic modulus of the particulate and metallic matrix, respectively. $V_p$ is the volume fraction of the particulates, $s$ is the particulate aspect ratio which is maintained as unity in this analysis. The term $q$ is used for mathematical simplification and is expressed in Equation 7.

$$q = \frac{(E_p/E_m - 1)}{(E_p/E_m) + 2s} \quad (7)$$

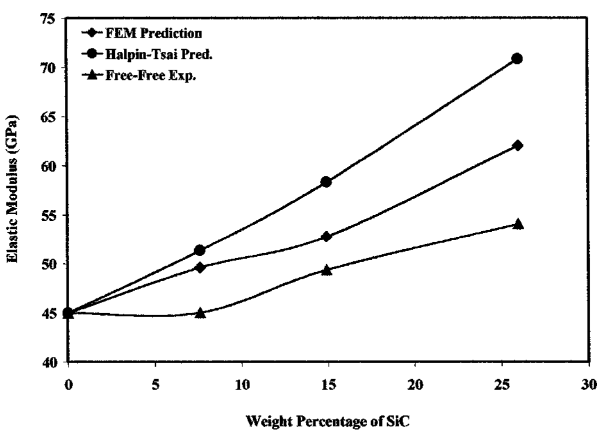


*Figure 5* Variation of elastic modulus with weight percentage of SiC particulates.

## 6. Discussion

The free-free experimental test method was found to be highly repeatable and had many advantages for testing the dynamic modulus. The method was non-destructive, and used low cost instrumentation and accommodates a wide variety of specimen configurations and sizes. Testing by the impulse excitation technique was exceptionally simple and fast, and the results are not affected much by human error. The as extruded bars were tested by the impulse excitation method without any additional machining as compared to a tensile test method.

From Table I, an increase in the elastic modulus of the material with an increase in weight percentage of the SiC particulates can be observed for both theoretical predictions and experimental results. This increase in the elastic modulus may be attributed to the higher elastic modulus of SiC, which is reported to be about 450 GPa [16] as compared to the ductile magnesium metal matrix's $E$ value of 45 GPa [15].

The elastic modulus prediction using FEM shown in Fig. 5 matches close to the Halpin-Tsai predicted elastic modulus $E_{H\text{-}T}$ results. The advantage of FEM over these empirical equations is that it can model various types of particulate shapes, dispersion characteristics and material. The experimentally determined elastic modulus values for composite samples are found to be lower when compared to the FEM results for all the three composite samples. Based on static modulus experiments [20] for aluminium based metal matrix composites, it has been observed that presence of clusters and voids in the MMC significantly decreases the magnitude of elastic modulus. The same can be expected to hold good for the dynamic modulus experimental results obtained in the present study. Further work is continuing in this area. Secondly, in the present FEM model, weight percentage of SiC was varied by changing the size of the SiC particulate while keeping the number of particulates constant. This was contrary in the case of the actual sample, where the size of the particulates is maintained constant while the number of particulates is varied to achieve a particular volume fraction of SiC. This discrepancy between the FEM model and the experimental sample is expected to have an influence on the final results shown in Fig. 5 since the mean spacing distance of the particulates has more bearing on the bending stiffness and on the mode shape of the beam. Thirdly, an ideal interface between the SiC and the Mg-matrix was assumed in the FEM model and the presence of clusters was not taken into account, as seen in Fig. 2. While further efforts are being made to circumvent these shortcomings, the results of the modeling, however, follow the similar trend as predicted by the Halpin-Tsai equation and are comparatively in better agreement with the experimentally determined values.

## 7. Conclusions

The following inferences can be made from the analysis:

1. The free-free beam type flexural resonance method can successfully be used to measure the dynamic modulus of the Mg-SiC composites.

2. The elastic modulus values obtained using finite element method were found to be in better agreement with the experimental modulus values when compared to those predicted by the Halpin-Tsai method. Both the theoretical approaches, in common, exhibit the increasing trend of elastic modulus value with an increase in weight percentage of SiC particulates.

3. An increase in the weight percentage of SiC particulates in the magnesium matrix increases the elastic modulus of the composite material.

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